

Problem 2.27

[Difficulty: 5]

2.27 Using the data of Problem 2.26, find and plot the streakline shape produced after the first second of flow.

Given: Velocity field

Find: Plot streakline for first second of flow

Solution:

Following the discussion leading up to Eq. 2.10, we first find equations for the pathlines in form

$$x_p(t) = x(t, x_0, y_0, t_0) \quad \text{and} \quad y_p(t) = y(t, x_0, y_0, t_0)$$

where x_0, y_0 is the position of the particle at $t = t_0$, and re-interpret the results as streaklines

$$x_{st}(t_0) = x(t, x_0, y_0, t_0) \quad \text{and} \quad y_{st}(t_0) = y(t, x_0, y_0, t_0)$$

which gives the streakline at t , where x_0, y_0 is the point at which dye is released (t_0 is varied from 0 to t)

For particle paths, first find $x(t)$ $\frac{dx}{dt} = u = u_0$

Separating variables and integrating

$$dx = u_0 \cdot dt \quad \int_0^t \quad x = x_0 + u_0 \cdot (t - t_0)$$

For $y(t)$ we have

$$\frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right] \quad \text{so} \quad \frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left[t - \frac{x_0 + u_0 \cdot (t - t_0)}{u_0}\right]\right]$$

and

$$\frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \frac{x_0}{u_0}\right)\right]$$

Separating variables and integrating

$$dy = v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \frac{x_0}{u_0}\right)\right] \cdot dt \quad y = y_0 + v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \frac{x_0}{u_0}\right)\right] \cdot (t - t_0)$$

The streakline is then

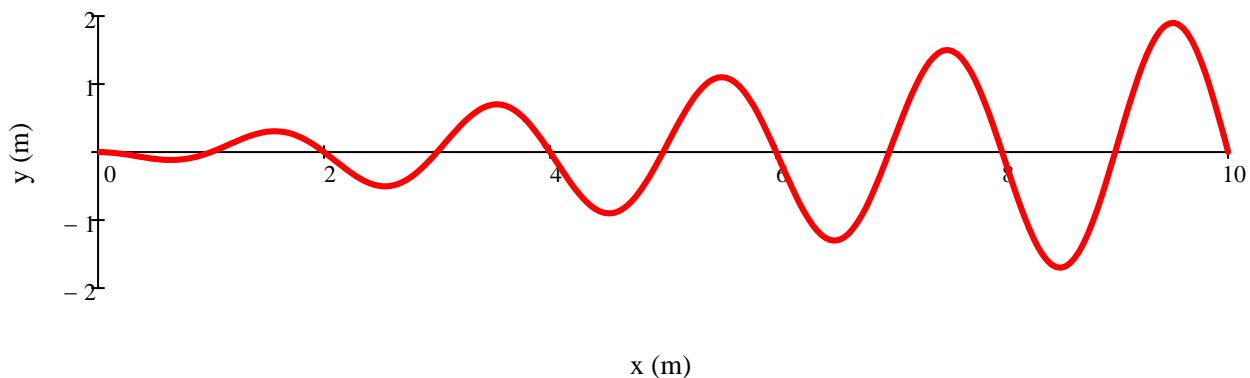
$$x_{st}(t_0) = x_0 + u_0(t - t_0) \quad y_{st}(t_0) = y_0 + v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \frac{x_0}{u_0}\right)\right] \cdot (t - t_0)$$

With

$$x_0 = y_0 = 0$$

$$x_{st}(t_0) = u_0 \cdot (t - t_0) \quad y_{st}(t_0) = v_0 \cdot \sin[\omega \cdot (t_0)] \cdot (t - t_0)$$

Streakline for First Second



This curve can be plotted in *Excel*. For $t = 1$, t_0 ranges from 0 to t .